Experiment 14: Standing Waves on a String

I. About the Experiment

A. Background

A wave on a string is a transverse disturbance that is transmitted along the length of the string. The vibration at any point on the string is at right angles to the direction of transmission. That is why the wave is referred to as a transverse wave. The speed of transmission depends on the tension in the string and the mass of the string. The lighter the string, the faster the wave. The higher the tension, the faster the wave.

When the transverse wave reaches the end of the string, it is usually reflected backwards. If the end of the string is fixed in place, the reflection is inverted . If the end of the string is free to move up and down, the reflected wave is not inverted. When a reflected wave encounters another incoming wave, they pass through each other and continue onward. At points of overlap, the combined amplitude is just the sum of the amplitudes of the waves.

If the incoming wave is a continuous train of sine waves, the overlap with the reflected waves can produce interesting patterns on the string. Most of these patterns travel with time. But under certain special conditions, these patterns become standing waves. These wave patterns do not travel on the string. Instead they stay in place and vibrate up and down with no movement along the string. These standing waves are important because they are an example of a phenomenon that occurs in many places in nature. Our hearing and sight are based on standing waves in sound and light.

There are three conditions needed to produce standing waves. There must be waves traveling in both directions, which is caused on a string by the incoming wave and its reflection. The two waves must have the same frequency, which is automatic when one wave is a reflection of the incoming wave. And finally, the length of the string must be exactly a multiple of a half wavelength of the wave. When all three conditions are met, standing waves result.

B. Theory

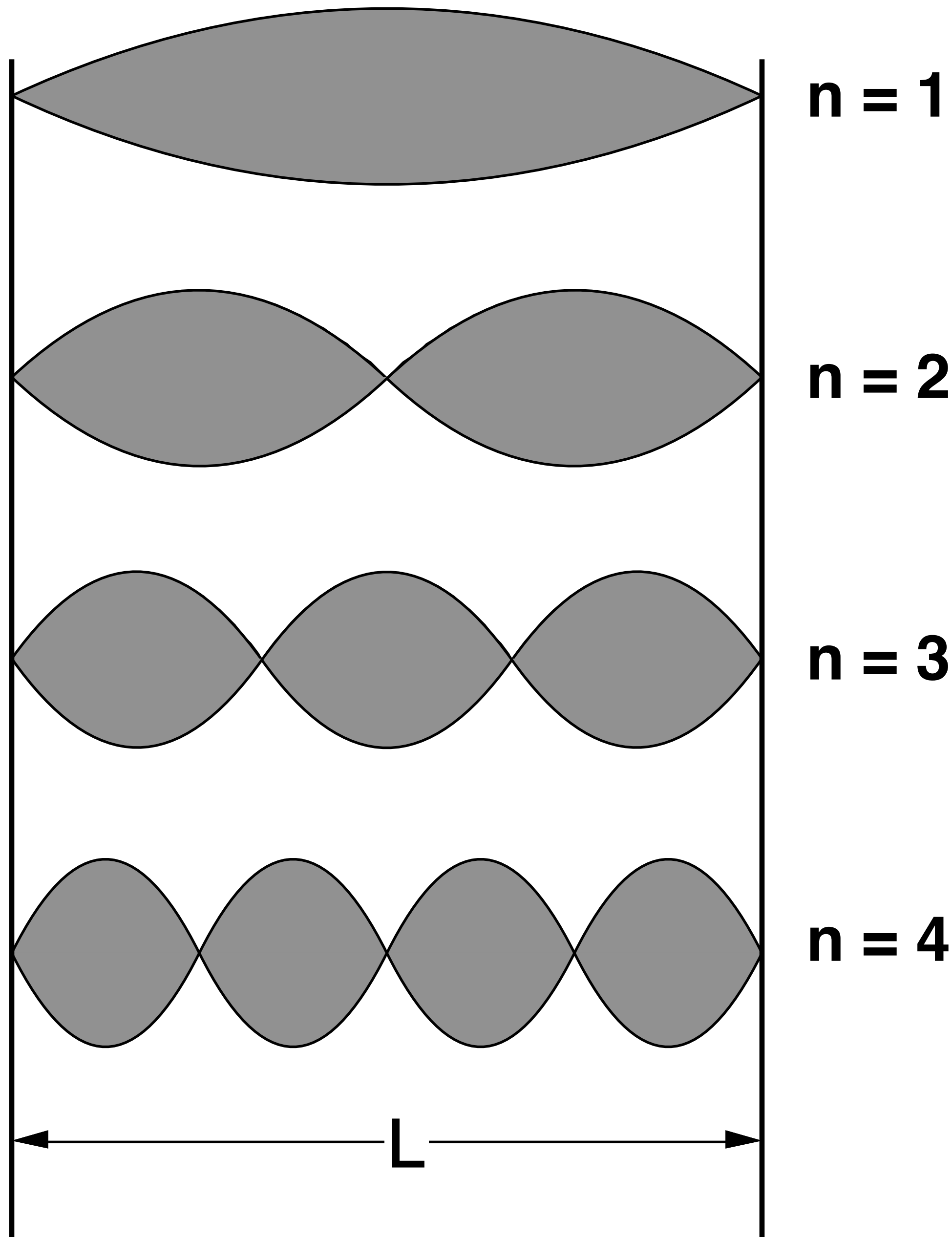
It can be shown that the speed of a wave on a taut string is:

$$v = (T/\mu)^{1/2} \qquad \text{Eqn. 1}$$

where v is the velocity of the wave, T is the tension in the string, and μ is the mass per unit length of the string. Combine this with the basic equation for waves:

$$v = f\lambda \qquad \text{Eqn. 2}$$

where v is the velocity of the wave, f is the frequency of the wave, and λ is the wavelength of



the wave. This gives the wavelength for a given frequency:

$$\lambda = v/f = (T/\mu f^2)^{1/2} \qquad \text{Eqn. 3}$$

This gives the wavelength for a wave depending on the tension, mass per unit length, and frequency. But the wavelength is not dependent on the length of the string. The third condition for standing waves is that the length of the string must be exactly a multiple of a half wavelength of the wave. So the condition for standing waves on a string can be expressed mathematically as:

$$L = n(\lambda/2) = n(T/4\mu f^2)^{1/2} \qquad \text{Eqn. 4}$$

where L is the length of the string and n is the number of half wavelengths standing on the string. Standing waves only occur when this condition is met. If any of the factors above is varied a small amount, the standing waves will decrease in amplitude and a small traveling wave will develop. If the variation is increased, the standing waves will diminish to nothing and a larger traveling wave will develop. During the experiment you will see that even a 1% variation in the tension will produce a substantial decrease in the size of the standing waves.

II. The Experiment

The standing waves experiment can be done in many ways. You could vary the length of the string, its mass, or the driving frequency. However, the easy way is to keep all of these constant and vary the tension of the string by adding weights to a hanging mass at the end of the string. So we will need to rewrite Equation 4 to solve for tension:

$$T = 4\mu f^2 L^2/n^2 \qquad \text{Eqn. 5}$$

In the experiment you will use a vibration frequency of 60 cycles per second (60 Hertz). You will measure the length of the string L as the distance from the vibrator end to the center of the pulley.

As you vary the tension by reducing weight on the weight hanger, the patterns on the string will change. Starting at a high tension you will be able to produce a single half wavelength. The number will increase with decreased tension. Decreasing the tension will decrease the wave speed and thus the wavelength. So the number of half wavelengths standing on the string will increase as they shorten with decreased tension. You will find that it is difficult to get more than 4 standing half wavelengths on the string as the patterns become unstable at low string tensions.

So the experiment consists of finding the mass needed to produce the proper tension to get 1, 2, 3, and 4 standing half wavelengths on the string. You will then calculate the theoretical value of the tension for each case using Equation 5. Finally you will compare your experimental value of T to your theoretical value and express the difference as a % error. You may find it helpful to put a few sheets of black paper behind the string to get a better view. When you are close to a standing wave, pulling up or down slightly on the hanging mass will suggest whether to add or subtract mass. It may also help to hold the pulley wheel when you are close to a standing wave.

III. Procedure

1. Measure the length of the string from the vibrator end to the center of the pulley.
   String Length L ______________

2. Adjust the mass on the weight hanger to produce the maximum amplitude for each standing wave pattern. You can start at a high mass and examine the single half wavelength case first or you could start at low hanging mass and go higher. Note that the higher half wavelength cases have points of zero motion called nodes.

   Number of half wavelengths ____1____ Hanging Mass ___________________

   Number of half wavelengths ____2____ Hanging Mass ___________________

   Number of half wavelengths ____3____ Hanging Mass ___________________

   Number of half wavelengths ____4____ Hanging Mass ___________________

3. Return the hanging mass to that for the single half wavelength case as seen above. Now examine the amplitude of the vibration. Measure the approximate amplitude of the vibration at the vibrator end. Then measure the much larger amplitude at the middle of the standing half wave. This maximum point is called an antinode.

   Amplitude at vibrator end _______ Amplitude at antinode _____________

IV. Calculations and Analysis

1. Convert the hanging mass into string tension using T = mg. Calculate the value of μ for each case. Then find the average value of μ from your data. Use this to calculate the % difference from the average μ for each half wavelength.

Half wavelengths	Experimental Tension (mg)	Calculated value of μ	% Difference
1			
2			
3			
4			
	Average μ ---->		

2. Calculate the ratio of the amplitudes in part III.3. (Antinode amplitude/Vibrator amplitude). This is called the quality factor. The square of the quality factor represents the amount of energy stored to energy input per cycle. Quality Factor _____________________

V. Questions

1. In this experiment, the standing wave pattern is changed by changing the wave speed. If the wave speed was not changed, describe another way to change the standing wave patterns?

2. There have been some spectacular bridge failures due to standing waves set up by vibrations. Using the ideas of standing waves and quality factor, explain why a column of marching soldiers is ordered to walk across a bridge rather than march across.