Experiment 9: Conservation of Momentum

I. About the Experiment

In this experiment you will investigate collisions between carts on a dynamic cart track. On these tracks the frictional forces between the cart and the tracks is extremely small. The principles used to analyze these collisions are Newton's second and third laws, which are used to develop the principle of conservation of momentum.

In this experiment two carts move on a horizontal track with no horizontal forces except the forces they exert on each other during a collision. Let the masses of the carts be m_1 and m_2, and their velocities $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ Both $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are functions of time, since they change during the collision. While the carts are in contact during the collision, they exert time dependent forces on each other. Let these forces on m_1 and m_2 be $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. Now from Newton's second law,

$$\vec{\mathbf{F}}_1 = m_1 \frac{d\vec{\mathbf{v}}_1}{dt} \quad \text{and} \quad \vec{\mathbf{F}}_2 = m_2 \frac{d\vec{\mathbf{v}}_2}{dt} \qquad \text{Eqn.1}$$

Newton's third law states that the two interaction forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. have equal magnitude but opposite direction, which means we can write

$$\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2 \qquad \text{Eqn.2}$$

Combining Equation 2 with Equation 1 we obtain

$$m_1 \frac{d\vec{\mathbf{v}}_1}{dt} + m_2 \frac{d\vec{\mathbf{v}}_2}{dt} = 0$$

which may be rearranged

$$\frac{d}{dt}\left(m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2\right) = 0 \qquad \text{Eqn.3}$$

The quantity $m_1\vec{\mathbf{v}}_1$ is defined to be the momentum of the first cart, usually denoted by $\vec{\mathbf{p}}_1$, and similarly for the other cart. Thus, Equation 3 states that the total momentum $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2$ does not change during the collision, in as much as its time derivative is always zero at each instant of time during the collision. Thus

$$m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{ a constant} \qquad \text{Eqn.4}$$

This is a very powerful result, in as much as it makes no detailed assumptions about the forces, which may vary in a complicated way during the collision. The result is often expressed by saying that the momentum is <u>conserved</u> in the collision, which means the momentum is the same after the collision as it was before the collision. Of course, the principle is valid only when there are no horizontal forces on the carts other than their mutual forces. (Why?) It is important to note that the statement that momentum is conserved does not necessarily mean that kinetic energy is conserved. As we shall see, kinetic energy may or may not be conserved, depending on the kind of collision.

It is customary to classify collisions according to the relative velocity of the two bodies before and after the collision. If the relative velocity has the same magnitude before and after, the collision is said to be perfectly elastic and the total kinetic energy is the same after the collision as it was before the collision. If the relative velocity has smaller magnitude after than before, the collision is semi-elastic, and if it is zero after the collision (i.e., if the two bodies stick together), it is completely inelastic. The ratio of final to initial relative velocities is called the coefficient of restitution, denoted by e. For a perfectly elastic collision e = 1, for a completely inelastic one e = 0, and for a semi-elastic collision e is between 0 and 1.

Since the motion takes place along a straight line, we may dispense with vector notation (remember, though, that the velocities are signed quantities). In the setup used in this experiment, mass m_1 is given a velocity **before** (b) the collision v_{1b}, and mass m_2 is initially at rest ($v_{2b} = 0$). Let the velocities **after** (a) the collision be v_{1a} and v_{2a}. Then by definition the coefficient of restitution is

$$e = \frac{v_{2a} - v_{1a}}{v_{1b} - v_{2b}} \quad \text{and since } v_{2b} = 0 \quad e = \frac{v_{2a} - v_{1a}}{v_{1b}} \qquad \text{Eqn.5}$$

If m_2 is larger than m_1, v_{1a} may be negative, but the relative velocity after the collision is still given by $v_{2a} - v_{1a}$.

In the setup used in this experiment the statement of conservation of momentum (Equation 4) reduces to

$$m_1 v_{1b} = m_1 v_{1a} + m_2 v_{2a} \qquad \text{Eqn.6}$$

We note that the ratio of the kinetic energy after the collision to that of the kinetic energy before the collision is given by

$$R = \frac{\frac{1}{2} m_1 v_{1a}^2 + \frac{1}{2} m_2 v_{2a}^2}{\frac{1}{2} m_1 v_{1b}^2} \qquad \text{Eqn.7}$$

II. The Experimental Set-up

The Dynamic Carts and Tracks

The track has leveling screws at one end. It also has bumpers at each end to prevent the carts from rolling off. Note that these bumpers have strong magnets in them which interact with magnets in the carts. The carts have low friction wheel bearings. One cart has a spring plunger in it and is referred to as the plunger cart. The other cart doesn't have a plunger and is referred to as the collision cart. Both carts can hold a fin that passes through the photo-gate and momentarily blocks the gates light beam. The upper part of the fin is 2 cm long and the lower part of the fin is 5 cm long. Only use the upper (2 cm) portion of the fin in these experimental procedures. Also, only use the fin with the plunger cart.

By creating collisions of various types between carts of equal and unequal masses we can draw certain conclusions concerning momentum transfer. Using the electronic timing units and the photogates we can determine the velocities of the two carts before and after collisions. From the velocities and the masses of the carts we can determine their momenta and kinetic energies.

A. Cautions in Using the Dynamic Carts

1. Keep floppy discs away from the magnets in the bumpers and carts.

2. Do not use excessive velocities with the carts. A gentle rolling motion will yield better results.

B. Operation of the Dynamic Carts

1. Familiarize yourself with the operation of the carts by observing the motion and collision interactions between the plunger cart and collision cart. NOTE: The plunger is not used in this experiment. To keep it from interfering with the collision cart, push it all the way in and pull up slightly to engage the plunger latch. You should practice both elastic and inelastic collisions.

2. Place either cart in the center of the track and adjust the leveling screws to keep the cart from drifting.

C. The Electronic Timing Unit and Photo-gates

This system consists of an electronic timer and two photo-gates. The timer operates in several modes. For this experiment**, only the GATE mode is used**. When operating in the gate mode, the unit senses when a photo-gate is blocked and records how long this blockage lasts. This is called an event. The speed of the cart is determined by dividing the length of the fin (2 cm) by the time of the event. Hence, if the upper part of the fin passes through a photo-gate and the timing unit displays 0.25 seconds for the event, then the cart traveled at 8 cm/sec. The timing unit also has a memory feature that allows you to record two events. To use it, place the switch labeled off/t1/t2 in the t1 position. Notice that the green memory LED indicator lights up. Press reset to clear the counter. The time for the first event is shown on the display while the total time for both events is stored in memory. Write down the value for the first event, then press the switch to the t2 position to display the total time. Next, subtract the value for t1 from the total time shown as t2. The result is the time for event 2.

Practice timing both elastic and inelastic collisions to familiarize yourself with the equipment before conducting the actual experiment. Note that there is a red LED on each photo-gate that turns on when the light beam is blocked. Use this to help you adjust the height of the gate with respect to the cart fins.

III. The Experimental Procedure

A. Equal Masses - Elastic Collision

1. Carefully match the mass of the two carts by using a pan balance and adding mass to one of the two carts.
2. Place the collision cart at rest in the center of the track. Place one photo-gate in front of the collision cart. Set the height of this gate such that the 2 cm top section of the fin on the plunger cart interacts with the photo-gate light beam. Place the second photo-gate to measure the velocity after the collision.
3. Roll the plunger cart through the photo-gate and into the collision cart such that the internal cart magnets repel each other. Use the memory function to measure the velocity of the plunger cart both before and after the collision.
4. If you work carefully you will find that the fractional change in velocity between cart 1 and cart 2 is less than 5%. The percentage fractional change is given by

$$\left[\frac{v_{2a} - v_{1b}}{v_{1b}}\right] \times 100\%$$

Before proceeding with part B make sure that your measurements are within this tolerance.

Data Table

Part A: Cart Mass m = ____________

time for cart 1 t_1 = _______ distance for cart 1 d_1 = _______

time for cart 2 t_2 = _______ distance for cart 2 d_2 = _______

Part B: Cart Mass m = ____________

time for cart 1 t_1 = _______ distance for cart 1 d_1 = _______

time for combination t_2 = _______ distance for combination d_2 = _______

Part C: Cart 1 Mass m_1 = ____________ Cart 2 Mass m_2 = ____________

initial time for cart 1 t_1 = _______ initial distance for cart 1 d_1 = _______

final time for cart 1 t_1 = _______ distance for cart 1 d_1 = _______

time for cart 2 t_2 = _______ distance for cart 2 d_2 = _______

B. Equal Masses - Inelastic Collision

1. Turn the plunger cart around such that the Velcro will stick it to the collision cart.

2. Place the collision cart at rest in the center of the track. Place one photo-gate in front of the collision cart and the other photo-gate behind the collision cart. Adjust both gate heights for the 2 cm portion of the fin on the plunger cart.

3. Roll the plunger cart through the photo-gate and into the collision cart such that the Velcro sticks both carts to each other. Use the memory function to measure the velocity of the plunger cart before the collision and the velocity of both carts stuck together after the collision.

4. If you work carefully you will find that the fractional change in velocity between cart 1 and twice the velocity of the combined carts is less than 10%. In this case the percentage fractional change is given by

$$\left[\frac{2v_{2a} - v_{1b}}{v_{1b}}\right] \times 100\%$$

Before proceeding with part C make sure that your measurements are within this tolerance.

C. Unequal Masses

1. Add 200 gm of mass to the plunger cart. Before going on to part 2, you might like to see what happens when the second cart is initially in motion and the first is motionless.

2. For the case where $m_1 > m_2$ and the first cart is initially in motion, make quantitative measurements as in section A above. Use the same setup (but add a third photo-gate) as part B above and make the measurements the same way. That is, use the memory function to measure the velocity of the plunger cart both before and after the collision.

	A	B	C	D	E
1	Part A	Before		After	
2	mass (kg)	Plunger Cart	Collision Cart	Plunger Cart	Collision Cart
3	distance (m)	0.02	0	0	0.02
4	time (s)				
5	velocity (m/s)				
6	momentum (P) kg•m/s)				
7	kinetic energy (KE) (j)				
8					
9	Total P before			Total P after	
10	Total KE before			Total KE after	
11	% change in total P				
12	% change in toal KE				
13	Coeff. of Restitution				

Figure 1

IV. Calculations and Analysis For Parts A, B and C:

Note: if you lay out your spreadsheet with some care, after setting up the spreadsheet for Part A you can simply select the entire Part A region of the spreadsheet, copy this region and then paste it elsewhere. You can then enter the data for part B into this new region and the calculations will be redone with the new data. Repeat the copy and paste for Part C. Refer to Figure 1

1. From your measured velocities and the masses of the carts, **use an Excel spreadsheet to calculate** the momentum of each cart both before and after the collision and the total momentum before and after collision. Make sure to compare (% difference) the calculated total momentum after the collision with the total momentum before the collision.

2. **On the same Excel spreadsheet calculate** the kinetic energy of each cart before and after collision and the total K.E. before and after collision. Show these on your Excel table to show your results. Make sure to compare the calculated total K.E. after the collision with the total K.E. before the collision.

3. Calculate the coefficient of restitution for each trial and include it in your **spreadsheet** .

4. What conclusions can you draw concerning momentum transfer?

5. What conclusions can you draw concerning energy transfer? If energy is lost, what happens to it?

IV. Questions

1. Give an example of an elastic and an inelastic collision; State whether or not momentum and kinetic energy are conserved in each case.

2. When a tennis ball is thrown against a wall it appears to bounce back with exactly the same speed as it struck the wall. Is momentum conserved for this collision? Explain!

3. A person is standing on a **completely frictionless** surface. Can they walk on this surface? Explain! What could they do to move across the surface? Explain!

4. When a car comes to a normal stop **its** momentum is not conserved. Does this violate the principle of conservation of momentum? Explain! What happens to the car's kinetic energy?

5. Suppose we could place a small explosive charge on one of the bumpers so that it would explode on contact, pushing the two carts apart. Will momentum still be conserved? Explain. Will kinetic energy be conserved? What value will the coefficient of restitution have?

6. Suppose there are three astronauts outside their spaceship and that two of them decide to play catch with the third. This means the first two astronauts are attempting to throw the third back and forth. All astronauts weigh the same on earth and are equally strong. The first throws the second and the game begins. Describe the motion of the astronauts as the game proceeds. How long will the game last (i.e. how many throws) ?