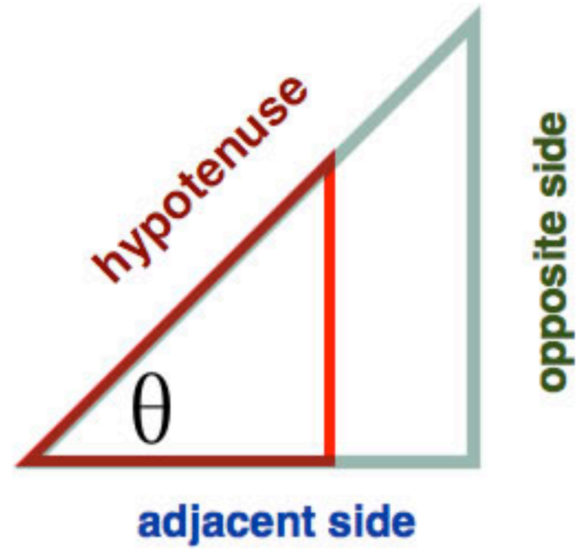
Trigonometry

*is the mathematics the deals with the **ratios** of the **lengths** of the sides of **similar right triangles**.*



Can you see the relationships between similar sides of the red and green triangle?

The two triangles are similar, their respective sides are in proportion to one another.

There are only six possible ratios with any right triangle. Three of which are

$$\frac{\text{opposite side}}{\text{hypotenuse}} \qquad \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \frac{\text{opposite side}}{\text{adjacent side}}$$

Each of these ratios is a function of the angle θ we can name them

$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

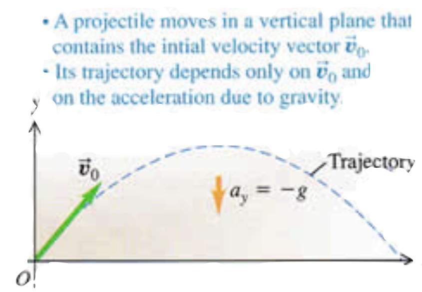
3.3 Projectile Motion

ActivPhysics 3.1–3.7

A **projectile** is any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, and a package dropped from an airplane are all examples of projectiles. The path followed by a projectile is called its *trajectory*.

To analyze this common type of motion, we'll start with an idealized model. We represent the projectile as a single particle with an acceleration (due to the earth's gravitational pull) that is constant in both magnitude and direction. We'll neglect the effects of air resistance and the curvature and rotation of the earth. Like all models, this one has limitations. The curvature of the earth has to be considered in the flight of long-range ballistic missiles, and air resistance is of crucial importance to a skydiver. Nevertheless, we can learn a lot by analyzing this simple model.

We first notice that projectile motion is always confined to a vertical plane determined by the direction of the initial velocity. We'll call this plane the x-y *coordinate plane*, with the x axis horizontal and the y axis directed vertically upward. Figure 3.9 shows a view of this plane from the side, along with a typical trajectory.



▲ **FIGURE 3.9** The trajectory of a projectile.

The key to analyzing projectile motion is the fact that *we can treat the x and y coordinates separately.* Why is this so? Anticipating a relation that we'll study in detail in Chapter 4, we note that the instantaneous acceleration of an object is proportional to (and in the same direction as) the net *force* acting on the object. Because of the assumptions made in our model, the only force acting on the projectile is the earth's gravitational attraction; we assume that this is constant in magnitude and always vertically downward in direction. Thus the vertical component of acceleration is the same as if the projectile moved only in the y direction, as it did in Section 2.6. Figure 3.10 shows two trajectories; the vertical displacements of the two objects at any time are the same, even though their horizontal displacements are different.

We conclude that the x component of acceleration, a_x, is zero and the y component a_y is constant and equal in magnitude to the acceleration of free fall:

$$a_x = 0; \qquad a_y = -g = -9.80\ \text{m/s}^2.$$

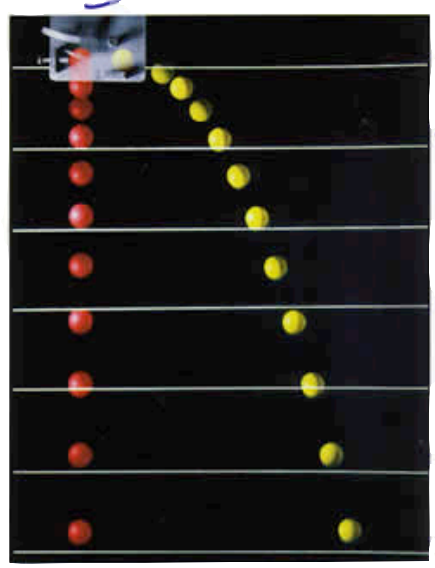
NOTE ▸ Remember that, by definition, g is always positive (because it is the magnitude of the acceleration vector due to gravity), but with our choice of coordinate directions, a_y is negative. ◂

So we can think of projectile motion as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration. We can then express all the vector relationships in terms of separate equations for the horizontal and vertical components. The actual motion is a combination of of these separate motions. Figure 3.11 shows the horizontal and vertical components of motion for a projectile that starts at (or passes through) the origin of coordinates at time $t = 0$. As in Figure 3.10, the projectile is shown at equal time intervals.

The horizontal (x) and vertical (y) components of $\vec{a}$ for a projectile are

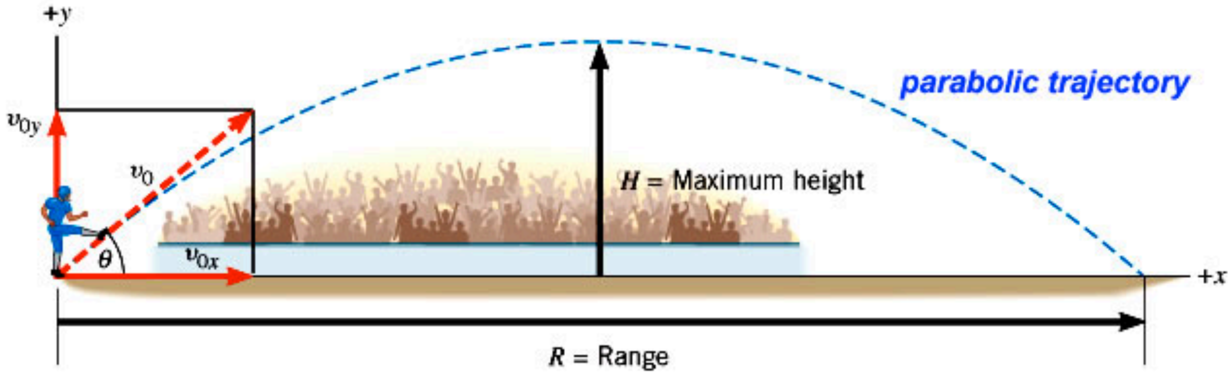
$$a_x = 0, \qquad a_y = -g.$$

We'll usually use $g = 9.80\ \text{m/s}^2$, but occasionally we'll use $g = 10\ \text{m/s}^2$ for approximate calculations.



▲ **FIGURE 3.10** Independence of horizontal and vertical motion: At any given time, both balls have the same y position, velocity, and acceleration, despite having different x positions and velocities. Successive images are separated by equal time intervals.

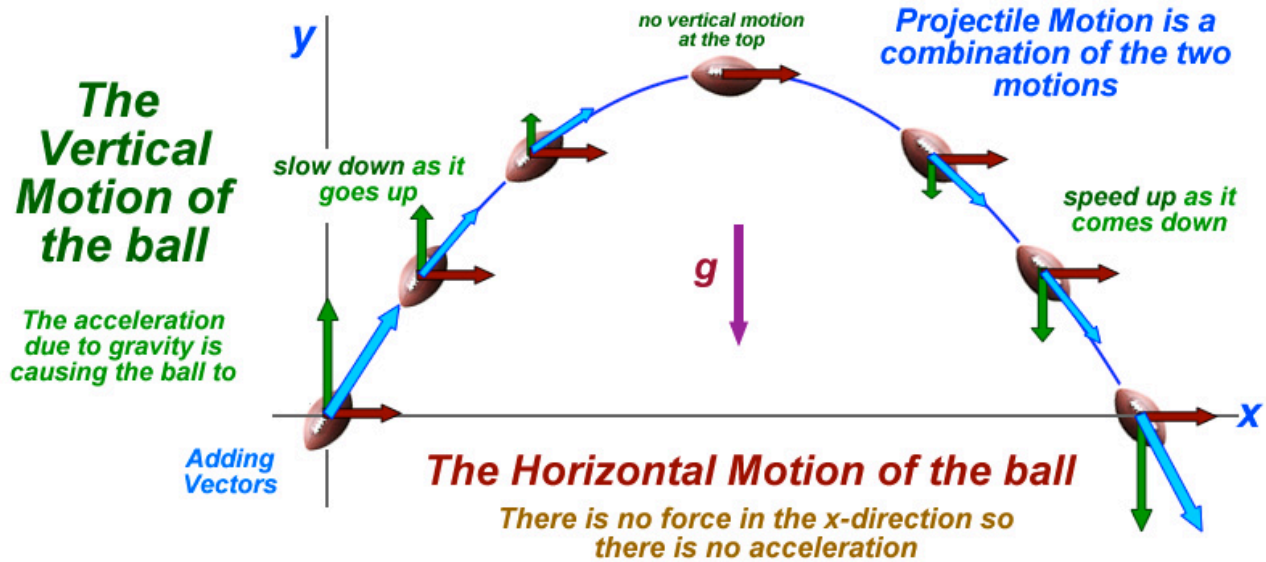
Projectile Motion - Kicking or Throwing a Football



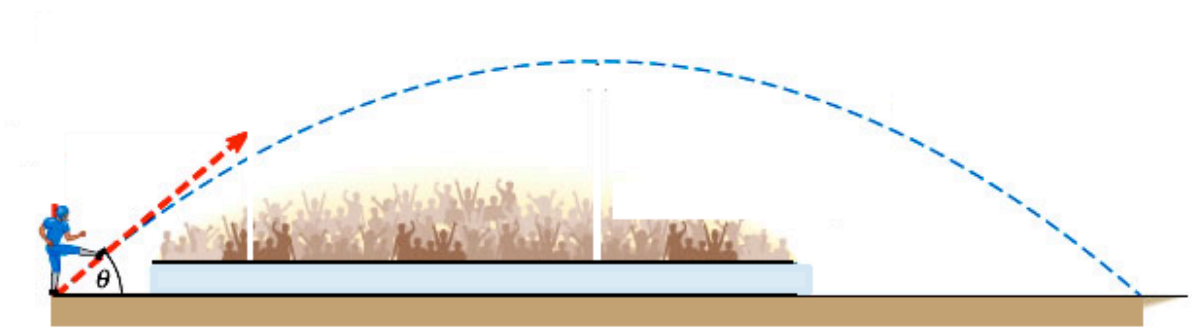
There are two independent motions to consider

*The motion in the **x-direction** and the motion in the **y-direction**.*

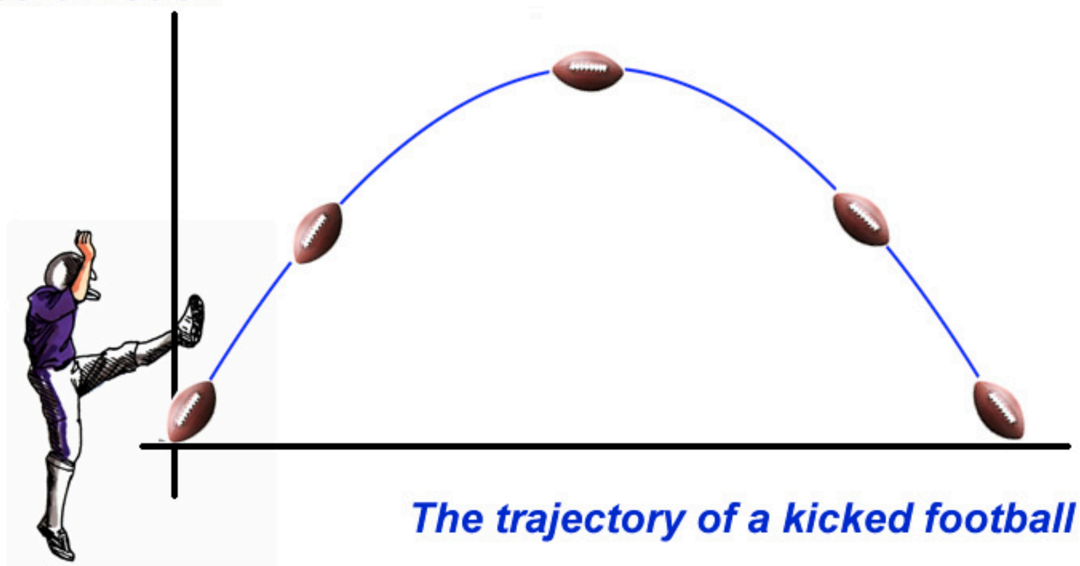
Projectile Motion - A Vector Perspective



Projectile Motion - Kicking or Throwing a Football



Projectile Motion



The trajectory of a kicked football

motion equations in one dimension

general form

$$v_f = v_o + at$$

$$v_f^2 = v_o^2 + 2a\Delta x$$

$$\boxed{\Delta x = v_o t} + \tfrac{1}{2}at^2$$

$$\Delta x = \tfrac{1}{2}(v_f + v_o)t$$

freefall

$$v_{fy} = v_{ox} - gt$$

$$v_{fy}^2 = v_{oy}^2 - 2g\Delta y$$

$$\Delta y = v_{oy} t - \tfrac{1}{2}gt^2$$

motion equations in two dimensions

y - direction

$$v_{fy} = v_{oy} - gt$$

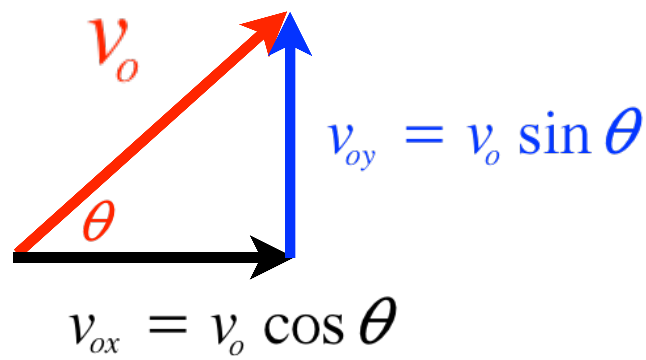
$$v_{fy}^2 = v_{oy}^2 - 2g\Delta y$$

$$\Delta y = v_{oy} t - \tfrac{1}{2}gt^2$$

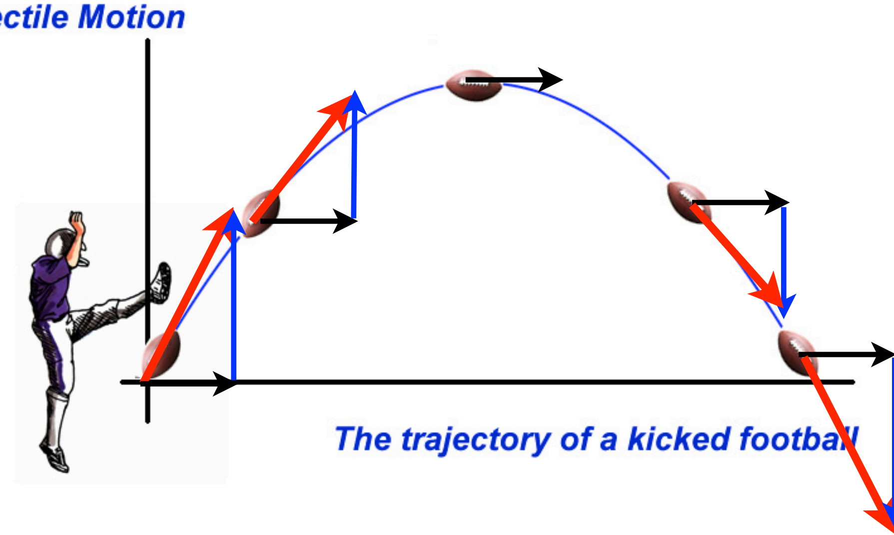
x - direction

no accleration in the x -direction

$$\Delta x = v_o t$$



Projectile Motion



The trajectory of a kicked football

motion equations in one dimension

$$v_f = v_o + at$$

$$v_f^2 = v_o^2 + 2a\Delta x$$

$$\Delta x = v_o t + \tfrac{1}{2}at^2$$

$$\Delta x = \tfrac{1}{2}(v_f + v_o)t$$

motion equations in two dimensions

$$v_{fy} = v_{oy} - gt$$

$$v_{fy}^2 = v_{oy}^2 - 2g\Delta y$$

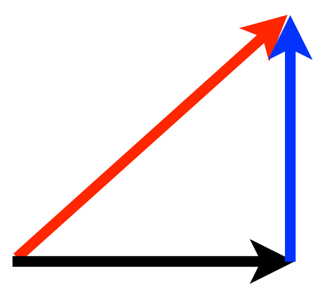
$$\Delta y = v_{oy} t - \tfrac{1}{2}gt^2$$

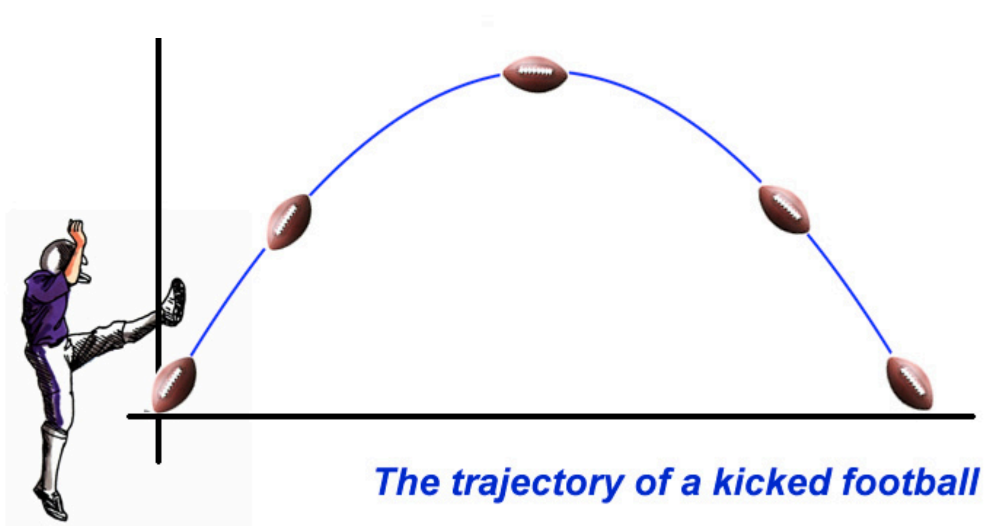
$$v_{fy} = v_{ox} - gt$$

$$v_{fy}^2 = v_{oy}^2 - 2g\Delta y$$

$$\Delta y = v_{oy} t - \tfrac{1}{2}gt^2$$

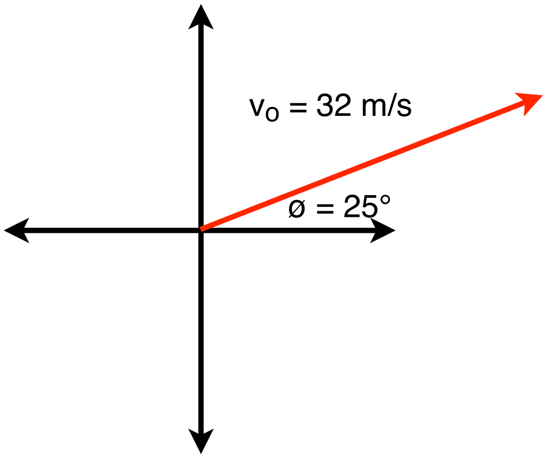
$$\Delta x = v_o t$$

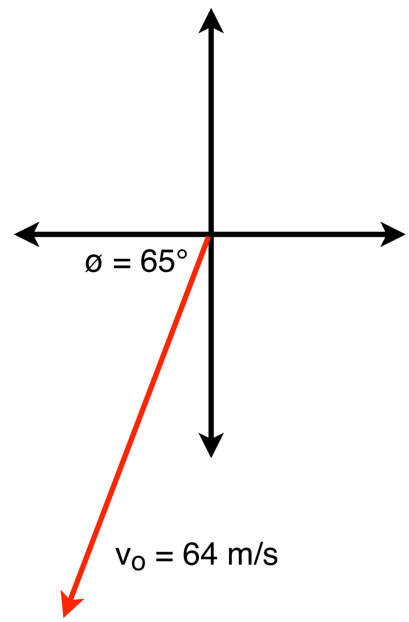


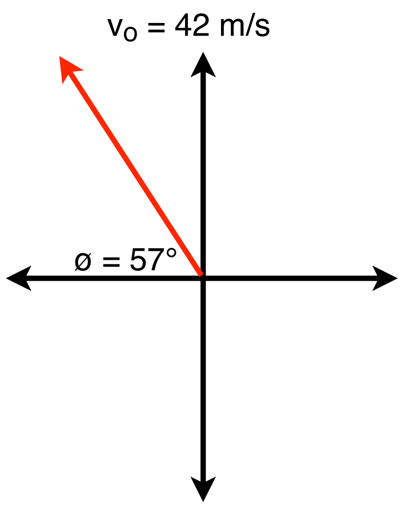


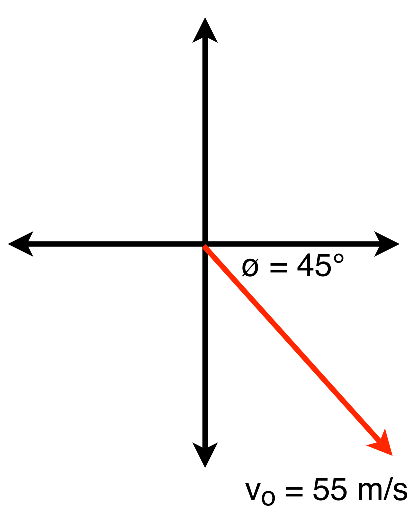
The trajectory of a kicked football

Review: Calculate and draw the components of the velocity vectors given. Use color.









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